

Latest Supernova data in the framework of the Generalized Chaplygin Gas model

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2 February 2008

ABSTRACT

We use the most recent Type-Ia Supernova data in order to study the dark energy - dark matter unification approach in the context of the Generalized Chaplygin Gas (GCG) model. Rather surprisingly, we find that data allow models with $\alpha > 1$. We have studied how the GCG adjusts flat and non-flat models, and our results show that GCG is consistent with flat case up to 68% confidence level. Actually this holds even if one relaxes the flat prior assumption. We have also analyzed what one should expect from a future experiment such as SNAP. We find that there is a degeneracy between the GCG model and a Λ CDM model with a phantom-like dark energy component.

Key words: Cosmology:Cosmological Parameters-Observations-Distance Scale-Supernovae type Ia-Method: Data Analysis.

1 INTRODUCTION

Recent cosmological observations reveal that the Universe is dominated by two invisible components. Type-Ia Supernova observations (Riess et al. 1998; Garnavich et al. 1998; Perlmutter et al. 1999), nucleosynthesis constraints (Burles et al. 2001), Cosmic Microwave Background Radiation (CMBR) power spectrum (Balbi et al. 2000; de Bernardis et al. 2000; Jaffe et al. 2001), large scale structure (Peacock et al. 2001) and, determinations of the matter density (Bachall & Fan 1998; Carlberg et al. 1998; Turner 2000) allow for a model where the clumpy component that traces matter, dark matter, amounts for about 23% of the cosmic energy budget, while an overall smoothly distributed component, dark energy, amounts for approximately 73% of the cosmic energy budget.

The most interesting feature of this dark energy component is that it has a negative pressure and drives the current accelerated expansion of the Universe (Riess et al. 1998; Garnavich et al. 1998; Perlmutter et al. 1999). From the theoretical side, great effort has been devoted to model dark energy. The most obvious candidate is the vacuum energy, an unanceled cosmological constant [see eg. Bento & Bertolami (1999), Bento et al. (2001)] for which $\omega_x \equiv p_x/\rho_x = -1$. Another possibility is a dynamical vacuum (Bronstein 1933; Bertolami 1986a; Bertolami 1986b; Ozer & Taha 1987) or quintessence. Quintessence models most often involve a single scalar field (Ratra & Peebles 1988a; Ratra & Peebles 1988b;

Wetterich 1988; Caldwell et al. 1998; Ferreira & Joyce 1998; Zlatev et al. 1999; Binétruy 1999; Kim 1999; Uzan 1999; Amendola 1999; Albrecht & Skordis 2000; Bertolami & Martins 2000; Banerjee & Pavón 2001a; Banerjee & Pavón 2001b; Sen & Sen 2001; Sen et al. 2001) or two coupled fields (Fujii 2000; Masiero et al. 2000; Bento et al. 2002a). In these models, the cosmic coincidence problem, that is, why did the dark energy start to dominate the cosmological evolution only fairly recently, has no satisfactory solution and some fine tuning is required.

More recently, it has been proposed that the evidence for a dark energy component might be explained by a change in the equation of state of the background fluid, with an exotic equation of state, the generalized Chaplygin gas (GCG) model, rather than by a cosmological constant or the dynamics of a scalar field rolling down a potential (Kamenshchik et al. 2001; Bilić et al. 2002; Bento et al. 2002b). In this proposal, one considers the evolution of the equation of state of the background fluid instead of a quintessence potential. The striking feature of this model is that it allows for an unification of dark energy and dark matter (Bento et al. 2002b). Moreover, it is shown that the GCG model may be accommodated within the standard structure formation scenario (Bilić et al. 2002; Bento et al. 2002b). Concerns about this point have been raised by Sandvik et al. (2002), however in this analysis, the effect of baryons has not been taken into account, which was shown to be important and allowing compatibility with the 2DF mass power spectrum (Beça et al. 2003). Also, the Sandvik et al. (2002) claim was based on the linear treatment of perturbations close to the present time, thus neglecting any non-linear effects.

Thus, given its potentialities, the GCG model has been the subject of great interest, and various attempts have been made to con-

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strain its parameters using the available observational data. Studies include Supernova data and power spectrum (Avelino et al. 2002), age of the Universe and strong lensing statistics (Dev et al. 2002), age of the Universe and Supernova data (Makler et al. 2002; Alcaniz et al. 2002). The tightest constraints were obtained by Bento et al. (2003a) using the CMBR power spectrum measurements from BOOMERANG (de Bernardis et al. 2002) and Archeops (Benoit et al. 2002), together with the SNe Ia constraints. It is shown that $0.74 \lesssim A_s \lesssim 0.85$, and $\alpha \lesssim 0.6$, ruling out the pure Chaplygin gas model. From the bound arising from the age of the APM 08279+5255 source, which is $A_s \gtrsim 0.81$ (Alcaniz et al. 2002), one can get tight constraints, namely $0.81 \lesssim A_s \lesssim 0.85$, and $0.2 \lesssim \alpha \lesssim 0.6$, which also rules out the Λ CDM model. These results were in agreement with the WMAP data (Bento et al. 2003b). It was also shown that the gravitational lensing statistics from future large surveys together with SN Ia data from SNAP will be able to place interesting constraints the parameters of GCG model (Silva & Bertolami 2003). As we shall see in Sections 3 and 4, all these constraints are consistent with Supernova data at 95% confidence level.

Recently Choudhury & Padmanabhan (2003) have analyzed the supernova data with currently available 194 data points [see also Padmanabhan & Choudhury (2003)] and shown that it yields relevant constraints on some cosmological parameters. In particular, it shows that when one considers the full supernova data set, it rules out the decelerating model with significant confidence level. They have also shown that one can measure the current value of the dark energy equation of state with higher accuracy and the data prefers the phantom kind of equation of state, $\omega_X < -1$ (Caldwell 2002). Moreover, the most significant observation of their analysis is that, without a flat prior, the latest Supernova data also rules out the preferred flat Λ CDM model which is consistent with other cosmological observations. In a previous paper, Alam et al. (2003) have reconstructed the equation of state of the dark energy component using the same set of Supernova data and found that the dark energy evolves rapidly from $\omega_x \simeq 0$ in the past to a strongly negative equation of state ($\omega_x \lesssim -1$) in the present, suggesting that Λ CDM may not be a good choice for dark energy. More recently, other groups have also analyzed these recent Supernova data in the context of different cosmological models for dark energy (Gong & Duan 2004; Nesseris & Perivolaropoulos 2004).

In this paper, we analyze the GCG model in the light of the latest supernova data (Tonry et al. 2003; Barris et al. 2003). We consider both flat and non-flat models. Our analysis shows that the problem with the flat model, which has been discussed in Choudhury & Padmanabhan (2003), can be solved in the GCG model in a sense that flat GCG model is consistent with the latest Supernova data even without a flat prior. We have also analyzed the confidence contours for a GCG model, that one expects from a future experiment such as SNAP. We find that there is a degeneracy between the GCG model and a XCDM model with a phantom-like dark energy component.

This paper is organized as follows. In Section 2 we discuss various aspects of the generalized Chaplygin gas model and its theoretical underlying assumptions. In Section 3 we describe our best fit analysis of the most recent supernova data in the context of generalized Chaplygin gas model. Section 4 contains our analysis for expected SNAP results. Finally, in Section 5 we present our conclusions.

2 GENERALIZED CHAPLYGIN GAS MODEL

The generalized Chaplygin gas (GCG) is characterized by the equation of state

$$p_{ch} = -\frac{A}{\rho_{ch}^\alpha}, \quad (1)$$

where A and α are positive constant. For $\alpha = 1$ the equation of state is reduced to so-called Chaplygin gas scenario first studied in cosmological context by Kamenshchik et al. (2001). Inserting the above equation of state in the energy conservation equation, one can integrate it to obtain (Bento et al. 2002b)

$$\rho_{ch} = \rho_{ch0} \left(A_s + \frac{(1 - A_s)}{a^{3(1+\alpha)}} \right)^{1/(1+\alpha)}, \quad (2)$$

where ρ_{ch0} is the present energy density of GCG and $A_s \equiv A/\rho_{ch0}^{(1+\alpha)}$.

One of the most striking features of this expression is that, the energy density of this GCG, ρ_{ch} , interpolates between a dust dominated phase, $\rho_{ch} \propto a^{-3}$, in the past and a de-Sitter phase, $\rho_{ch} = -p_{ch}$, at late times. This property makes the GCG model an interesting candidate for the unification of dark matter and dark energy. Indeed, it can be shown that the GCG model admits inhomogeneities and that, under the Zeldovich approximation, they evolve in a qualitatively similar fashion like the Λ CDM model (Bento et al. 2002b). Furthermore, this evolution is controlled by the homogeneous parameters of the model, namely, α and A .

There are several important aspects of the above equation which one should discuss before constraining the relevant parameters using Supernova data. Firstly, one can see from the above equation that A_s must lie in the range $0 \leq A_s \leq 1$. For $A_s = 0$, GCG behaves always as matter whereas for $A_s = 1$, it behaves always as a cosmological constant. Hence to use it as a unified candidate for dark matter and dark energy one has to exclude these two possibilities resulting the range for A_s as $0 < A_s < 1$.

To have an idea about the possible range for α , one has to consider the propagation of sound through this fluid. Given any Lagrangian $\mathcal{L}(X, \phi)$ for a field ϕ , where $X = \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$, the effective speed of sound entering the equations for the evolution of small fluctuations is given by

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} = \frac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}}. \quad (3)$$

Thus, for a standard scalar field model which has canonical kinetic energy term like $\mathcal{L} = X - V(\phi)$, the speed of sound is always equal to 1 irrespective of the equation of state. But for a Lagrangian containing a non-canonical kinetic energy term, one can have a sound speed quite different from 1. Actually, even $c_s^2 > 1$ is possible which physically means that the perturbations of the background fluid can travel faster than light as measure in the preferred frame where the background is homogeneous. For a time dependent background field, it does not lead to any violation of causality as the underlying theory is manifestly Lorentz invariant (Erickson et al. 2002).

For GCG it has been shown that the equation of state (1) can be obtained from a generalized version of the Born-Infeld action (Bento et al. 2002b)

$$\mathcal{L} = -A^{1/(1+\alpha)} \left[1 - (g^{\mu\nu}\phi_{,\mu}\phi_{,\nu})^{(1+\alpha)/2\alpha} \right]^{\alpha/(1+\alpha)}, \quad (4)$$

which for $\alpha = 1$ leads to the Born-Infeld action. If one computes c_s^2 for this action one can get using Eq. (2) the present value as $c_{s0}^2 = \alpha A_s$. As A_s is always positive, it restricts α to only positive

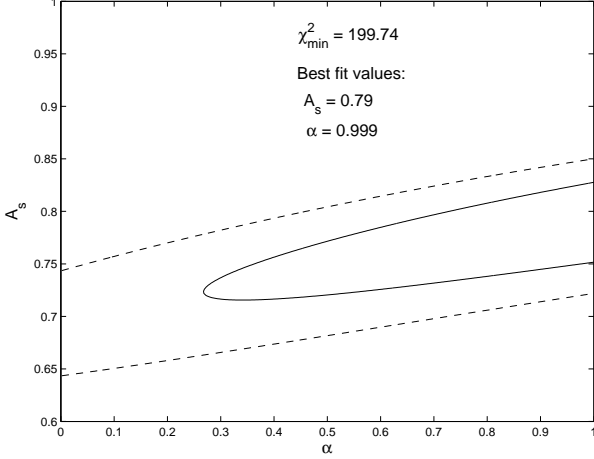


Figure 1. Confidence contours in the $\alpha - A_s$ parameter space for flat unified GCG model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively. The best fit value used for \mathcal{M}' is -0.033.

values. In all previous work, α has been restricted to a value up to 1. But one can see from the above expression for c_s^2 that as $0 < A_s < 1$, the maximum allowed value for α can be surely greater than 1 and that also depends on the value of A_s , e.g for $A_s = 0.5$, the allowed range for α is $0 \leq \alpha \leq 2$. Notice also that the dominant energy condition $\rho + p \geq 0$ is always valid in this case. Furthermore, there is no big rip in the future and asymptotically the Universe goes toward a de-Sitter phase.

Hence, on general grounds, restricting α up to 1 is not a very justified assumption. Moreover, this restriction arises mainly by considering the present day value of c_s^2 which is not an important epoch for structure formation. In general c_s^2 in this model is a time dependent quantity and in such cases it is not very proper to constrain α with the present day value of c_s^2 .

There is another reason for not restricting α up to 1. It has been shown by Kamenshchik et al. (2001) that one can also model the Chaplygin gas with a minimally coupled scalar field with canonical kinetic energy term in the Lagrangian density. Performing this exercise for the GCG, leads to a potential for this scalar field of the form

$$V = V_0 e^{3(\alpha-1)\phi} \left[\cosh\left(\frac{m\phi}{2}\right)^{2/(\alpha+1)} + \cosh\left(\frac{m\phi}{2}\right)^{-2\alpha/(\alpha+1)} \right] \quad (5)$$

where V_0 is a constant and $m = 3(\alpha+1)$. For $\alpha = 1$, one recovers the potential obtained by Kamenshchik et al. (2001). Now as we have discussed earlier, for a minimally coupled scalar field with a canonical kinetic energy term, the value of c_s^2 is always 1 irrespective of the equation of state. Hence if one considers this kind of scalar field to model GCG, there is no such restriction on α coming from the sound speed.

In what follows, we shall consider that A_s lies in the range $0 < A_s < 1$ and the only constrain on α that we shall consider is that it takes positive values. We should also point out that the $\alpha = 0$ case corresponds to the Λ CDM model.

The Friedmann equation for a non-flat unified GCG model in general is given by

$$H^2 = H_0^2 \left[\Omega_{ch} \left(A_s + (1 - A_s)(1+z)^{3(1+\alpha)} \right)^{1/(1+\alpha)} + \Omega_k (1+z)^2 \right], \quad (6)$$

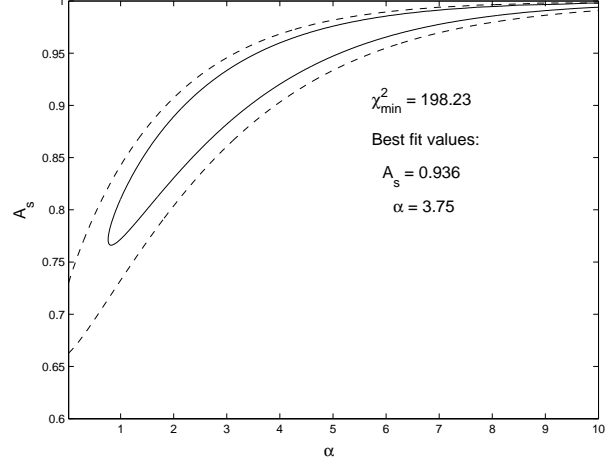


Figure 2. Same as Figure 1, but with a wider range for α .

where H_0 is the present day value of the Hubble constant. Ω_{ch} and Ω_k are the present day density parameters for GCG and the curvature. For a flat Universe $\Omega_k = 0$ and $\Omega_{ch} = 1$, whereas for the non-flat case $\Omega_{ch} = 1 - \Omega_k$.

3 RECENT SUPERNOVA DATA AND THE BEST FIT ANALYSIS

To perform the best fit analysis of our GCG model with the recent Supernova data, we follow the method discussed by Choudhury & Padmanabhan (2003). As far as the Supernova observation is concerned, the cosmologically relevant quantity is the *apparent magnitude*, m , given by

$$m(z) = \mathcal{M} + 5 \log_{10} D_L(z), \quad (7)$$

where $D_L = \frac{H_0}{c} d_L(z)$, is the dimensionless luminosity distance, and the luminosity distance $d_L(z)$ is given by $d_L(z) = r(z)(1+z)$ where $r(z)$ is the comoving distance. Also $\mathcal{M} = M + 5 \log_{10} \left(\frac{c/H_0}{1 \text{ Mpc}} \right) + 25$ where M is the absolute magnitude for the Supernova which is believed to be constant for all Type-Ia Supernova.

In our analysis, we take the 230 data points listed in Tonry et al. (2003) along with the more recent 23 points from Barris et al. (2003). Also, as discussed by Choudhury & Padmanabhan (2003), for low redshifts, data might be affected by the peculiar motions, making the measurements of the cosmological redshifts uncertain; hence we shall consider only those points with redshifts $z > 0.01$. Moreover, since it is difficult to be sure about the host galaxy extinction, A_v , we do not consider points which have $A_v > 0.5$. Hence in our final analysis, we consider only 194 points, which are similar to those considered by Choudhury & Padmanabhan (2003).

The Supernova data points given by Tonry et al. (2003) and Barris et al. (2003) are listed in terms of luminosity distance $\log_{10} d_L(z)$ together with the corresponding error $\sigma_{\log_{10} d_L(z)}$. These distances are obtained assuming some value of \mathcal{M} which may not be the true value. Hence, in our analysis we shall keep it as free parameter while fitting the data.

The best fit model is obtained by minimizing the quantity

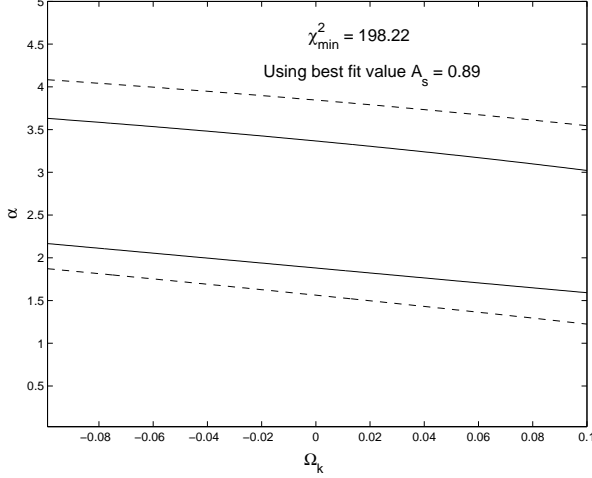


Figure 3. Confidence contours in the $\Omega_k - \alpha$ parameter space for non flat unified GCG model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively. The best fit value used for \mathcal{M}' is -0.033.

$$\chi^2 = \sum_{i=1}^{194} \left[\frac{\log_{10} d_{L\text{obs}}(z_i) - 0.2\mathcal{M}' - \log_{10} d_{L\text{th}}(z_i; c_\alpha)}{\sigma_{\log_{10} d_L}(z_i)} \right]^2 \quad (8)$$

where $\mathcal{M}' = \mathcal{M} - \mathcal{M}_{\text{obs}}$ is a free parameter denoting the difference between the actual \mathcal{M} and the assumed value \mathcal{M}_{obs} in the data. As discussed by Choudhury & Padmanabhan (2003), we have also taken into account the uncertainty arising because of the peculiar motion at low redshift by adding an uncertainty of $\Delta v = 500 \text{ km s}^{-1}$ to $\sigma_{\log_{10} d_L}(z)$,

$$\sigma_{\log_{10} d_L}(z) \rightarrow \sigma_{\log_{10} d_L}(z) + \left(\frac{1}{\ln 10} \frac{1}{d_L} \frac{\Delta v}{c} \right)^2. \quad (9)$$

This correction is more effective at low redshifts, i.e for small values of d_L .

In our subsequent best fit analysis, the minimization of (8) is done with respect to \mathcal{M}' , α , A_s and Ω_k . The parameter \mathcal{M}' is a model independent parameter and hence its best fit value should not depend on a specific model. We have checked that when minimizing (8) with respect to \mathcal{M}' , the best fit value for \mathcal{M}' for all of the models considered here is -0.033 which is also consistent with that obtained by Choudhury & Padmanabhan (2003). Hence, in our subsequent analysis, we shall use always this best fit value, $\mathcal{M}' = -0.033$.

3.1 Flat case

For this case, we assume $\Omega_k = 0$ and consider only two parameters, α and A_s . We first restrict α to be ≤ 1 . In Figure 1, we have shown the 68% and 95% confidence contours in $\alpha - A_s$ parameter space. The best fit values for $[\alpha, A_s]$ are given by $[0.999, 0.79]$. The best fit value of α is very close to its upper limit since the actual best fit value lies in the region beyond $\alpha = 1$. Also, up to 68% confidence level, the $\alpha = 0$, i.e. the Λ CDM case, is excluded although it is consistent at 95% confidence level.

Next we allow α to vary for a wider range. The Figure 2 represents the same as Figure 1 but with α taking a wider range. The best fit values for $[\alpha, A_s]$ are now $[3.75, 0.936]$. This high value of α might just be a statistical fluke though, as the confidence regions ex-

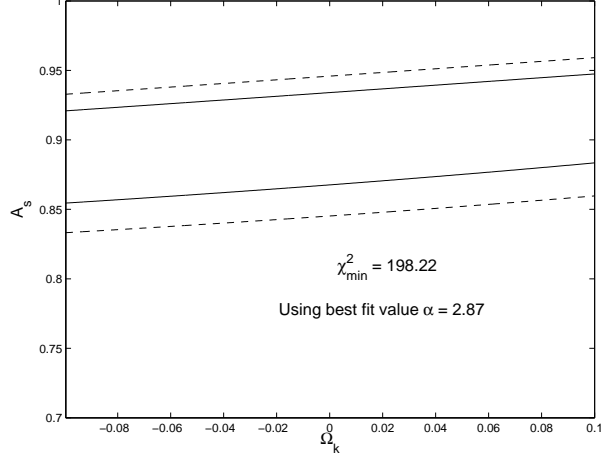


Figure 4. Same as Figure 3, but in the $\Omega_k - A_s$ parameter space.

hibit a very shallow valley along the α direction. Here, also at 68% confidence level, the $\alpha = 0$, Λ CDM case, is excluded, although it is consistent at 95% confidence level. It should be noted that both Choudhury & Padmanabhan (2003) and Tonry et al. (2003) also found some conflict between the Λ CDM model and the SN Ia data, namely, that when imposing a flat Universe prior, the data ruled out the vacuum energy as an allowed dark energy component, and actually favoured a phantom energy component. Here we see that the GCG model fits the data well, and, as mentioned previously, without the theoretical complications that plague the phantom energy model, namely the dominant energy condition is not broken, and there is no big rip singularity in the future.

It is also clear from the minimum value for χ^2 obtained in these two cases, that when one allows to vary α beyond 1, one obtains a better fit to the Supernova data.

3.2 Non-flat case

Another problem that the Λ CDM has with the new SN Ia data is that without a flat prior, a flat Λ CDM Universe has also been ruled out at 68% confidence (Choudhury & Padmanabhan 2003). Tonry et al. (2003) argued that this was probably due to some overlooked systematic error, since a small systematic error of 0.04 mag was able to make the flat Λ CDM consistent with the data. Here we attempt to find if the GCG model might alleviate this problem.

For this, we allow a non-vanishing curvature in our model. We now have three parameters in our model namely, α , A_s and the density parameter for the curvature at present, Ω_k . First we assume that our Universe deviates slightly from the flat model assuming Ω_k to vary between $[-0.1, 0.1]$. In this case the best fit values for α , A_s and Ω_k are $[2.87, 0.89, -0.099]$. It suggests that the data prefers a negative curvature. In Figure 3, we have shown the 68% and 95% confidence contours in the $\Omega_k - \alpha$ plane assuming the best fit value for A_s , whereas in Figure 4, we have shown the same contours in the $\Omega_k - A_s$ plane assuming the best fit value for α .

Both α and A_s are constrained significantly and 68% confidence limits on α and A_s are $[1.6, 3.625]$ and $[0.856, 0.946]$ respectively. It shows that the data prefers higher value for α and the Λ CDM limit ($\alpha = 0$) is excluded, both for 68% and 95% confidence limit. Also one can see from both figures that the flat case

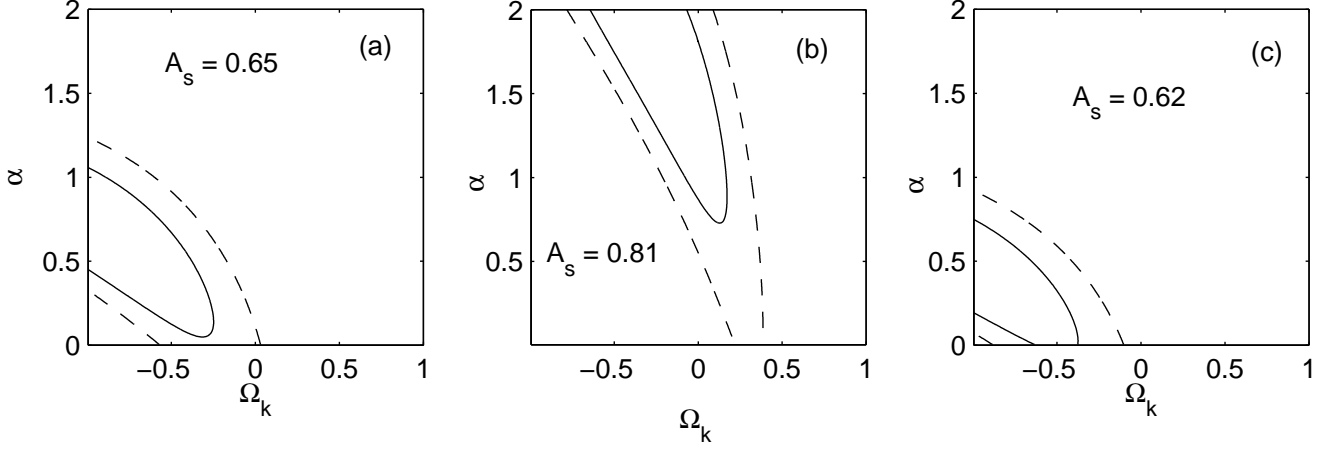


Figure 5. Confidence contours in the $\Omega_k - \alpha$ parameter space for non-flat unified GCG model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively. The best fit value used for \mathcal{M}' is -0.033. Figure (a) is for the best fit mean value of A_s , whereas (b) and (c) are for the values in the wings of 68% confidence limit.

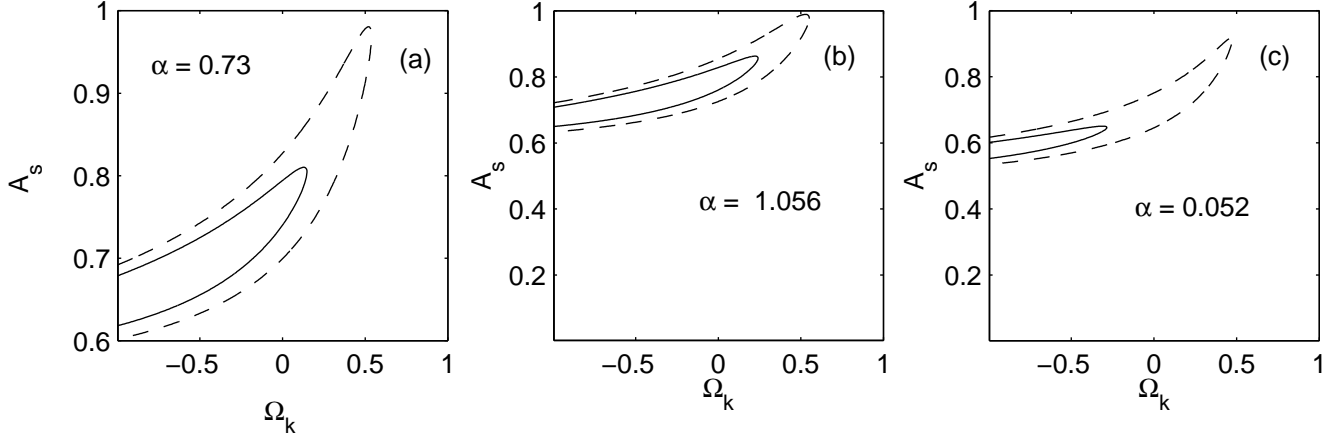


Figure 6. Same as Figure 5, but in the $\Omega_k - A_s$ parameter space. Figure (a) is for the best fit value of α whereas (b) and (c) are for the values in the wings of 68% confidence limit.

$\Omega_k = 0$ is consistent with the data for both 68% and 95% confidence level, but for a higher value of α .

We allow now more curvature in our model and consider the range for Ω_k to vary as $[-1, 1]$. In this case the best fit values for α , A_s and Ω_k are $[0.73, 0.65, -0.999]$ and the resulting χ^2_{min} is 197.99 showing that there is an improvement in the quality of fit. The 68% confidence limits on α and A_s are $[0.052, 1.056]$ and $[0.62, 0.81]$, respectively. Also the allowed range for α shifts more significantly towards smaller values and data do allow the model to get closer to Λ CDM ($\alpha = 0.052$) as well as to the Chaplygin gas model ($\alpha = 1$).

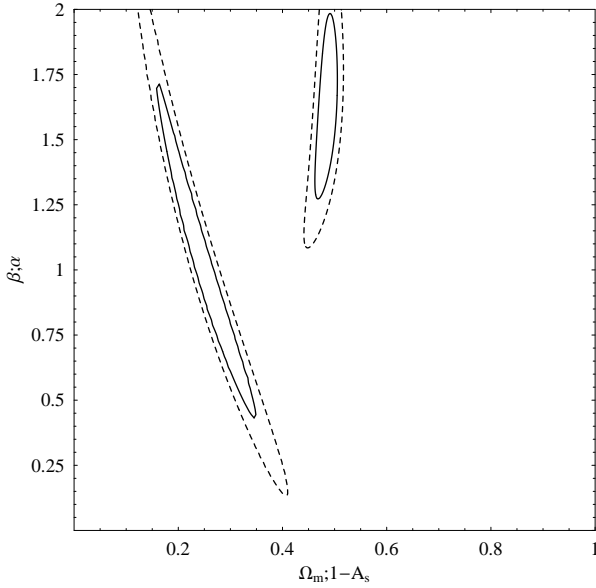
In Figure 5, we have shown the 68% and 95% confidence contours in $\Omega_k - \alpha$ plane assuming the best fit value for A_s , as well as for its values in the wings of 68% confidence limit. It shows that the allowed range is quite sensitive to the parameter A_s . For smaller values of A_s (but within its 68% confidence limit) the flat model ($\Omega_k = 0$) is more inconsistent with the data and negative curvature is preferred. But for higher values of A_s , e.g. $A_s = 0.81$ which

still falls within the 68% confidence limit, it shows that flat models as well as models with small but both positive and negative curvature, are allowed [consistent with WMAP bound on Ω_{total} , $0.96 < \Omega_{total} < 1.08$ (Spergel et al. 2003)] but for non zero values of α . It suggests that assuming a GCG model, one can alleviate the problem of consistency with a flat universe as pointed out by Choudhury & Padmanabhan (2003) for a Λ CDM model.

In Figure 6, we have shown the same contours but now in the $\Omega_k - A_s$ plane assuming the best values for α as well as its values at the wings of the 68% confidence limit. Figure 6(c) is for $\alpha = 0.052$ which is almost a Λ CDM model, and it is quite similar to what obtained by Choudhury & Padmanabhan (2003) for a Λ CDM model. It shows that model that behaves more like Λ CDM ($\alpha = 0.052$) is inconsistent with a flat universe at 68% confidence level. On the other hand, model that deviates more from the Λ CDM model, Figure 6(a) and (6b), is consistent with the flat universe with better confidence level. Like Figure 5, it again shows that even if one does

Table 1. SNAP specifications for a two year period of observations.

Redshift Interval	Number of SNe
0.0-0.2.....	50
0.2-1.2.....	1800
1.2-1.4.....	50
1.4-1.7.....	15

**Figure 7.** Expected confidence regions for SNAP for a GCG fiducial model with $1 - A_s = 0.25$ and $\alpha = 1$. The solid and dashed lines represents the 68% and 95% confidence regions respectively. The left contours are for GCG and the right ones are for XCDM. For GCG, the parameter space is $1 - A_s$ Vs. α whereas for XCDM, it is Ω_m Vs. β . We have marginalized over \mathcal{M} .

not take a flat prior, unlike the Λ CDM model, flat GCG model is consistent with the supernova data up to 68% confidence level.

4 EXPECTED SNAP CONFIDENCE REGIONS

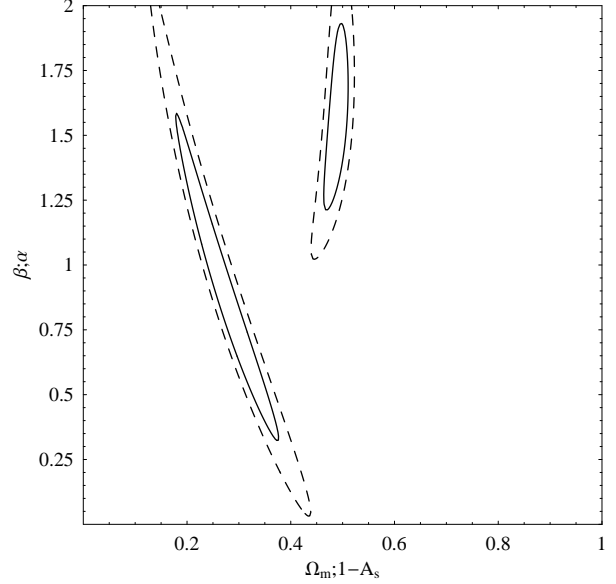
4.1 Method

To find the expected precision of a future experiment such as SNAP, one must assume a fiducial model, and then simulate the experiment assuming it as a reference model. This allows for estimates of the precision that the experiment might reach [see Silva & Bertolami (2003) and references therein for a more detailed description of the method employed here]. Let us then assume a fiducial model and functions χ^2 based on hypothetical magnitude measurements at the various redshifts. In this case,

$$\chi^2(\text{model}) = \sum_{z_i=0}^{z_{max}} \frac{[m_{\text{model}}(z_i) - m_{\text{fid}}(z_i)]^2}{\sigma^2(z)}, \quad (10)$$

where the sum is made over all redshift bins and $m(z)$ is as specified in Section 3.

Here together with the GCG model we also analyze a flat XCDM model (that is a Cold dark matter model together with a

**Figure 8.** Same as figure 7 but for a XCDM fiducial model with $\Omega_m = 0.49$ and $\beta = -\omega_x = 1.55$.

dark energy with an equation of state $p_x = \omega_x \rho_x$). We aim to show that if the Universe is indeed described by the GCG model, the fitting of a XCDM model to the data will reveal that the cosmic expansion is drawn by a phantom-like dark energy component.

To fully determine the χ^2 functions, the error estimates for SNAP must be defined. Following Weller & Albrecht (2002), we assume that the systematic errors for the apparent magnitude, m , are given by

$$\sigma_{sys} = \frac{0.02}{1.5} z, \quad (11)$$

which are measured in magnitudes such that at $z = 1.5$ the systematic error is 0.02 mag, while the statistical errors for m are estimated to be $\sigma_{sta} = 0.15$ mag. We place the supernovae in bins of width $\Delta z \approx 0.05$. We add both kinds of errors quadratically

$$\sigma_{mag}(z_i) = \sqrt{\sigma_{sys}^2(z_i) + \frac{\sigma_{sta}^2}{n_i}}, \quad (12)$$

where n_i is the number of supernovae in the i 'th redshift bin. The distribution of supernovae in each redshift bin is, as before, taken from Weller & Albrecht (2002), and shown in Table I.

Summarizing, for each fiducial model, the method used, consists in the following

- (i) Choose a fiducial model.
- (ii) Fit the XCDM model to the mock data, and obtain the respective confidence regions.
- (iii) Repeat the previous step to the GCG.

4.2 Results

In Figures 7 to 9 we show the confidence contours for the GCG and XCDM model for future SNAP observation taking different fiducial models. In all these Figures $\beta \equiv -\omega_x$. Also, along x-axis in all of these figures, we have plotted $1 - A_s$ (instead of A_s as

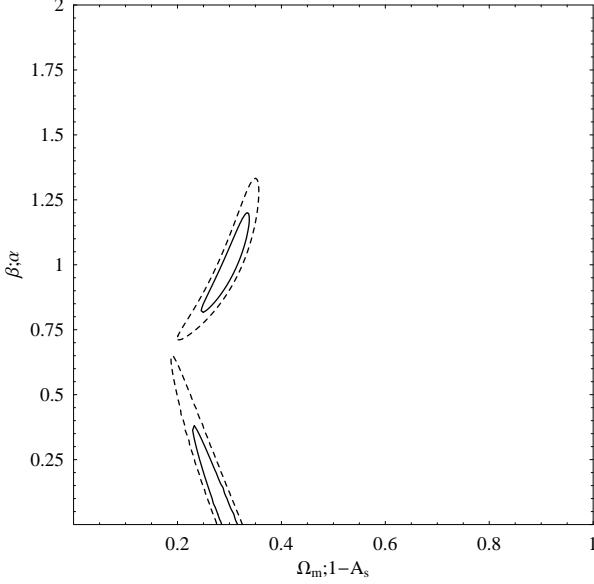


Figure 9. Same as figure 7, but for a Λ CDM fiducial model with $\Omega_m = 0.30$.

$1 - A_s$ represents Ω_m when $\alpha = 0$) for the GCG and Ω_m for the XCDM.

As mentioned above, our main aim is to explicitly show that a GCG Universe might appear as a XCDM Universe with a dark energy component that has a phantom-type equation of state. To do so, we have considered two fiducial models. The first corresponds to a Chaplygin model ($\alpha = 1$), with $1 - A_s = 0.25$. If one attempts to fit a XCDM model to the data (Figure 7), one finds that data favour a larger amount of matter than expected and a phantom-type dark energy component. This is fully consistent with Figure 13 of Tonry et al. (2003).

To further examine the degeneracy between models, in Figure 8 we have repeated the procedure assuming a XCDM fiducial model, with $\Omega_m = 0.49$ and $\beta = 1.55$ ($w = -1.55$). From examining Figures 7 and 8, one can see that both models appear essentially identical. Also for GCG, the confidence regions for both fiducial models are quite identical to what we have shown earlier in Figure 2 for the current Supernova data.

In Figure 9 we used a fiducial Λ CDM model, with $\Omega_m = 0.3$. As can be seen, the confidence regions are completely different from those found in the two previous cases. Also, for the GCG model the confidence regions are quite different from what we have earlier in Figure 2 for the current Supernova data, further hinting that indeed the Λ CDM model is not a good description of the Universe.

To illustrate the degeneracy between the GCG model and the XCDM model with a phantom like equation of state ($\omega_x < -1$), we Taylor expand the luminosity distance as,

$$d_L = \frac{c}{H_0} \left[z + \frac{(1 - q_0)}{2} z^2 - \frac{1}{6} (1 - q_0 - 3q_0^2 + j_0) z^3 + \dots \right] \quad (13)$$

where q_0 is the deceleration parameter related with the second derivative of the scale-factor and j_0 is the so-called jerk parameter (Visser 2003) related with the third derivative of the scale-factor. This is also one of the statefinder variables r proposed in Sahni et al. (2002). The subscript “0” means that quantities are evaluated at

present. The jerk parameter is related with the deceleration parameter q_0 as

$$j_0 = q_0 + 2q_0^2 + \frac{dq}{dz}|_0 \quad (14)$$

For the GCG model, one can calculate q_0 and $\frac{dq}{dz}|_0$ to get

$$\begin{aligned} q_0^{GCG} &= \frac{3}{2}(1 - A_s) - 1 \\ \frac{dq}{dz}|_0^{GCG} &= \frac{9}{2}A_s(1 - A_s)(1 + \alpha), \end{aligned} \quad (15)$$

whereas for XCDM model they turn out to be

$$\begin{aligned} q_0^{DE} &= \frac{3}{2}(1 - \omega_x(\Omega_m - 1)) - 1 \\ \frac{dq}{dz}|_0^{DE} &= \frac{9}{2}\omega_x^2(1 - \Omega_m)\Omega_m \end{aligned} \quad (16)$$

For the previous Supernova data obtained by Perlmutter et al. (1999) and Riess et al. (1998) for low redshifts ($z < 1$), it is sufficient to consider the first two terms in the series expansion of the luminosity distance $d_L(z)$ given above. In that case, one can see from the expression of q_0 for GCG, that Supernova data can only constrain A_s , as q_0 , for the GCG model, is independent of α . Moreover, in order to have degeneracy between the GCG and XCDM model, the q_0 parameter of these two models must be equal, which results that:

$$A_s = \omega_x(\Omega_m - 1) \quad (17)$$

Thus, if one gets a bound on A_s by fitting GCG model with low redshift Supernova data, then the same data can be fitted by a host of XCDM models (including Λ CDM model) provided the above equation is satisfied. Hence, for low redshifts ($z < 1$), GCG model is degenerate with all kind of different dark energy models with constant equation of state, including the Λ CDM model.

Now, as one goes to higher redshifts, which is the case for the current data that we are considering in this paper, one also has to consider the higher order terms in the series expansion of $d_L(z)$. As far the data we are studying in our paper, it is enough to consider terms up to order z^3 in the series expansion of d_L . Hence, in order to have a degeneracy between GCG and XCDM even for the high redshifts, the jerk parameter j_0 also has to be equal for the two models, which effectively means $\frac{dq}{dz}|_0$ has to be equal for the two models. Using this together with the equation (17), one finds that

$$\begin{aligned} \omega_x &= -\alpha(1 - A_s) - 1 \\ \Omega_m &= \frac{(1 + \alpha)(1 - A_s)}{1 + \alpha(1 - A_s)} \end{aligned} \quad (18)$$

In the above equation, ω_x and Ω_m are the equation of state and the density parameter of the XCDM model which is degenerate with a GCG model with the corresponding α and A_s parameters, for higher redshifts. Now one can see that for any GCG model ($\alpha > 0$), the corresponding equation state has to be always phantom type ($\omega_x < -1$) as $0 < A_s < 1$. This shows that although for the low redshift data, GCG model is degenerate with all kinds of constant equation of state dark energy model including the Λ CDM model, for higher redshift, the GCG is degenerate with a XCDM model but only with a phantom type of equation of state.

Finally, aiming to further illustrate the degeneracy of the GCG model with the XCDM model for large red-shifts, we plot in Figure 10 the behaviour of the dimensionless luminosity distance D_L as function of redshift for four different best fit models, namely: Λ CDM ($\omega_x = -1$, $\Omega_m = 0.3$), XCDM ($\omega_x = -2.07$, $\Omega_m = 0.51$), Chaplygin model ($\alpha = 1$, $A_s = 0.77$) and GCG ($\alpha = 3.75$,

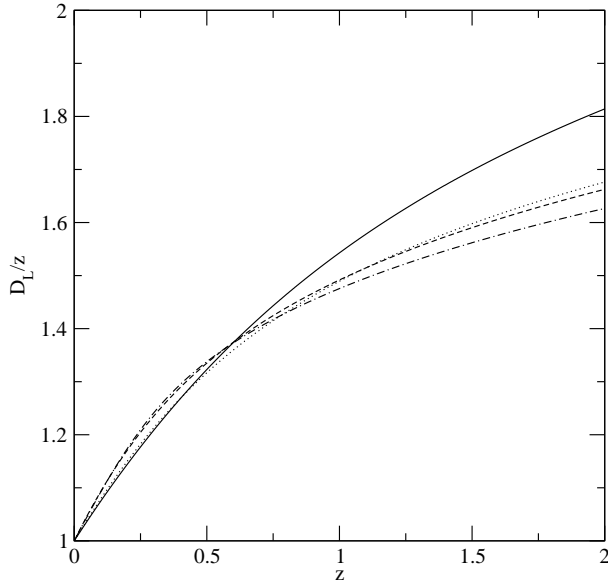


Figure 10. Dimensionless luminosity distance D_L/z as function of redshift for four different best fit models. The solid line is for Λ CDM. The dotted, dashed and dashed-dot lines are for Chaplygin, XCDM and GCG model, respectively. The values for the parameters of these models used in the plot are given in the main text.

$A_s = 0.936$) respectively. We have actually plotted the stretched D_L/z as function of redshift in order to graphically show the degeneracy.

5 CONCLUSIONS

We have analyzed the currently available 194 supernova data points within the framework of the generalized Chaplygin gas model, regarding GCG as a unified candidate for dark matter and dark energy. We have considered both, flat and non-flat cases, and used the best fit value for $\mathcal{M} = -0.033$ which is independent of a specific model, throughout our analysis. For the first time, we have crossed the $\alpha = 1$ limit for the GCG model and try to see whether the data actually allow it or not.

For the flat case, we have studied both cases, restricting α to the range $0 \leq \alpha \leq 1$ and also without any restriction on the upper limit α . From Figures 1 and 2, it is quite clear that data favours $\alpha > 1$, although there is a strong degeneracy in α . Also the quality of fit improves substantially as one relaxes the $\alpha = 1$ restriction. Moreover, the minimum values allowed for α and A_s at 68% confidence level are $[0.78, 0.778]$, which excludes the $\alpha = 0$, Λ CDM case, although there is no constrain on α at 95% confidence level.

Moreover, if one does not assume a flat prior for the analysis, our study shows that the flat GCG model for α values sufficiently different from zero, is consistent with the Supernova data up to 68% confidence level. It also allows small, both positive and negative curvature, making the GCG a somewhat better description than the Λ CDM model. This is consistent with recent result which shows that without a flat prior, a flat Λ CDM model, which is otherwise consistent with different cosmological observations, is not a good fit to the supernova data (Choudhury & Padmanabhan 2003). Moreover, the fact that GCG is a better fit to the Supernova data than Λ CDM, is consistent with the result of Alam et al. (2003),

who also have reconstructed a similar kind of evolving equation of state for the dark energy from the latest Supernova data.

We have also studied the confidence contours for a GCG and XCDM model expected from the future SNAP observation assuming different fiducial universes. In this regard, the degeneracy between the GCG model and a phantom-like dark energy scenario has made obvious in Section 4, where we have shown that when fitting a XCDM model to a GCG universe, the data will favour a phantom energy component, and vice-versa. This degeneracy is also illustrated analytically through the expression for the luminosity distance d_L as function of redshift. We have shown that for higher redshifts, GCG model is completely degenerate with a XCDM model with a phantom type of constant equation of state. We mention that it has already been noted in Maor et al. (2002) that time varying equations of state might be confused with phantom energy, and here we show that this is true for the GCG, without breaking the dominant energy condition and without a big rip singularity in the future. It should also be noted that with the exception of a cosmological constant, most dark energy models predict a time varying equation of state, therefore a constant dark energy equation of state might not be the best parametrization for dark energy. We have also shown that for a Λ CDM fiducial model, confidence regions for a GCG model, which are expected from future SNAP experiment, are quite different from what we have shown in Section 3.1, suggesting that SN Ia data does not favour the Λ CDM model.

Thus, our study shows that the generalized Chaplygin gas model is a very good fit to the latest Supernova data both with or without a flat prior. With future data, one expects the error bars to be reduced considerably, but we still expect that Supernova data will favour a generalized Chaplygin gas model with high confidence.

Acknowledgments

O.B. acknowledges the partial support of Fundação para a Ciência e a Tecnologia (Portugal) under the grant POCTI/FIS/36285/2000. The work of A.A.S. is financed by Fundação para a Ciência e a Tecnologia (Portugal) under the grant SFRH/BPD/12365/2003. The work of S.S. is financed by Fundação para a Ciência e a Tecnologia (Portugal) through CAAUL.

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